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INF07

MATEMATICKÉ ANALÝZY 1

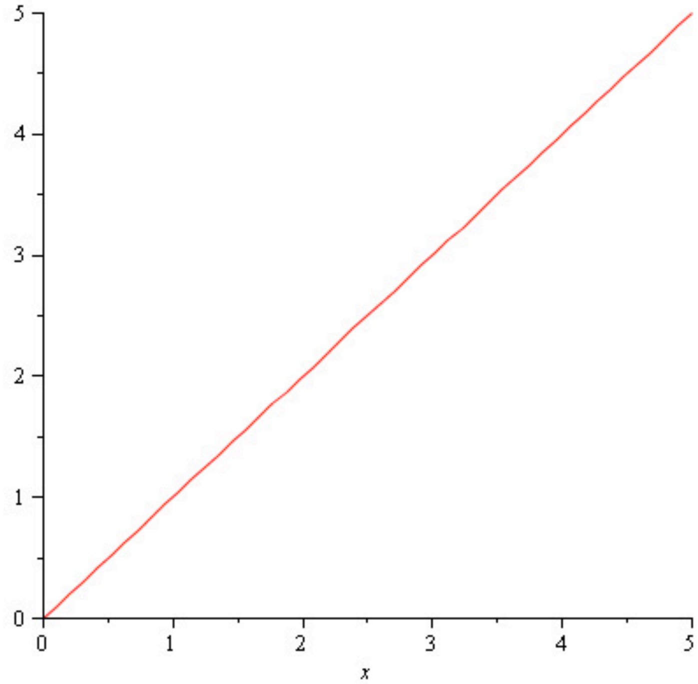
Téma:

Riemannův integrál

(Maple)

V Olomouci, dne 3.2.2008

$plot(x, x = 0..5)$



Počítejme Riemannův integrál pro funkci $f(x) = x$ na intervalu $\langle 0,5 \rangle$, tento interval rozdělíme na n podintervalů délky $h = \frac{b}{n}$ s dílčími body $x_i = hi = \frac{b}{n} i, i = 0 * n$. Jestliže v každém intervalu vezmeme za reprezentanta jeho levý (nebo pravý) krajní bod, pak dostaneme dva integrální součty S_l, S_r (formule levých a pravých obdélníku), pro které s použitím známé formule $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ dostaneme pro levé a pravé součty:

$$S_r(D_n, x) = \sum_{i=1}^{n} (ih) * b / n = \frac{b^2}{n^2} \sum_{i=1}^{n} i = \frac{b^2}{n^2} \frac{n(n+1)}{2} = b^2 (1 + \frac{1}{n}) / 2$$

$$S_l(D_n, x) = \sum_{i=0}^{n-1} (ih) * b / n = \frac{b^2}{n^2} \sum_{i=0}^{n-1} i = \frac{b^2}{n^2} \frac{(n-1)n}{2} = b^2 (1 - \frac{1}{n}) / 2$$

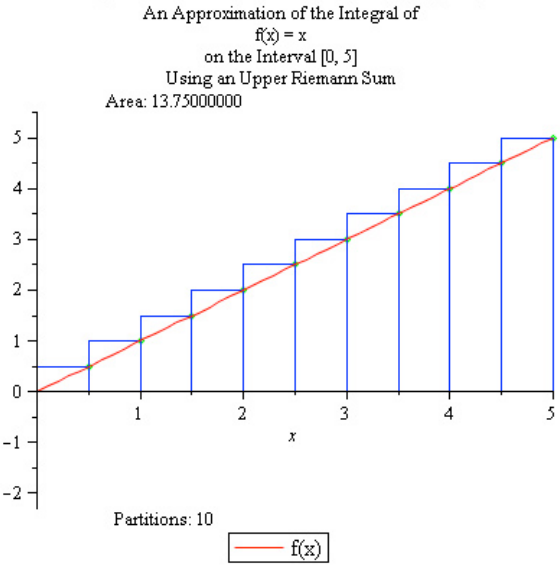
V případě intervalu $\langle 0,5 \rangle$ je b = 5, n je počet obdélníku pod grafem

$$\lim_{n \to \infty} \frac{5^2 (1 + \frac{1}{n})}{2} = \lim_{n \to \infty} \frac{25 + \frac{25}{n}}{2} = 12.5$$

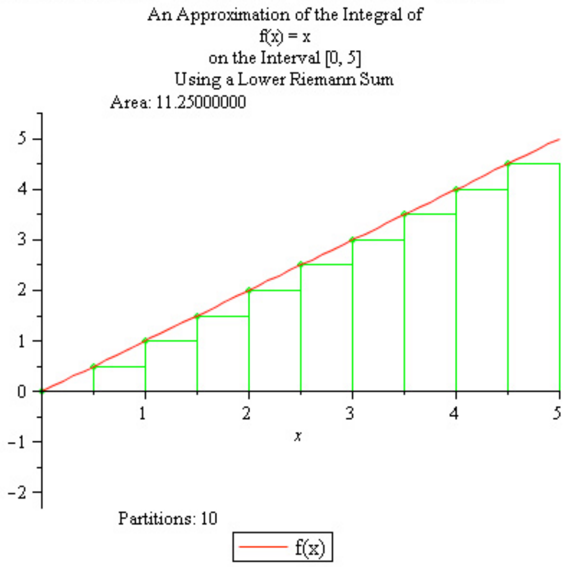
$$\lim_{n \to \infty} \frac{5^2 (1 - \frac{1}{n})}{2} = \lim_{n \to \infty} \frac{25 - \frac{25}{n}}{2} = 12.5$$

Graf pro n = 10

RiemannSum(*x*, *x* = 0 ..5, *method* = *upper*, *output* = *plot*);

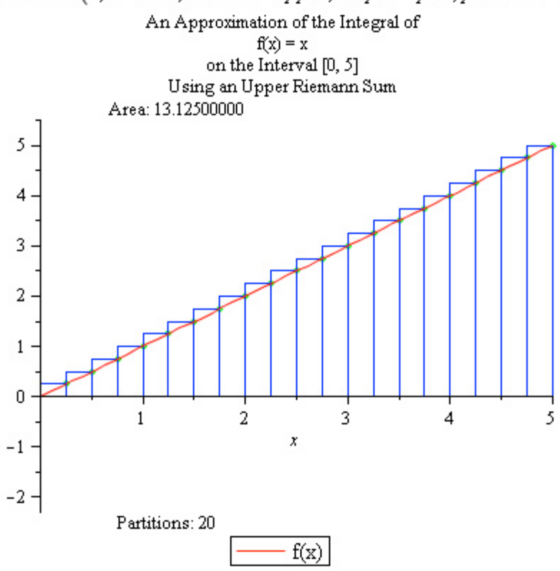


RiemannSum(*x*, *x* = 0 ..5, *method* = *lower*, *output* = *plot*);

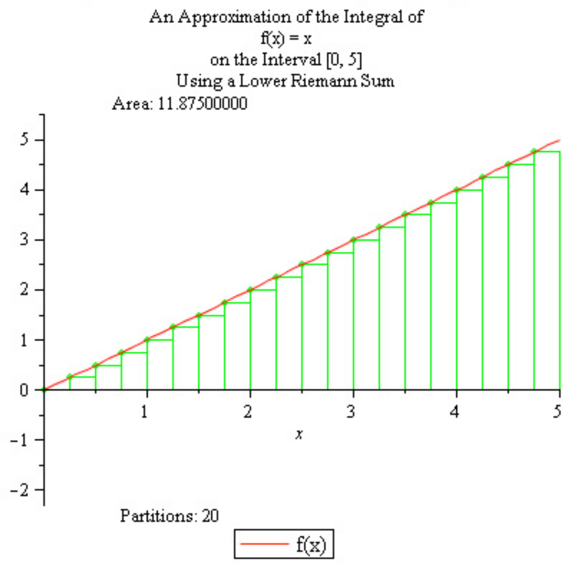


Graf pro n = 20

RiemannSum(*x*, *x* = 0 ..5, *method* = *upper*, *output* = *plot*, *partition* = 20);

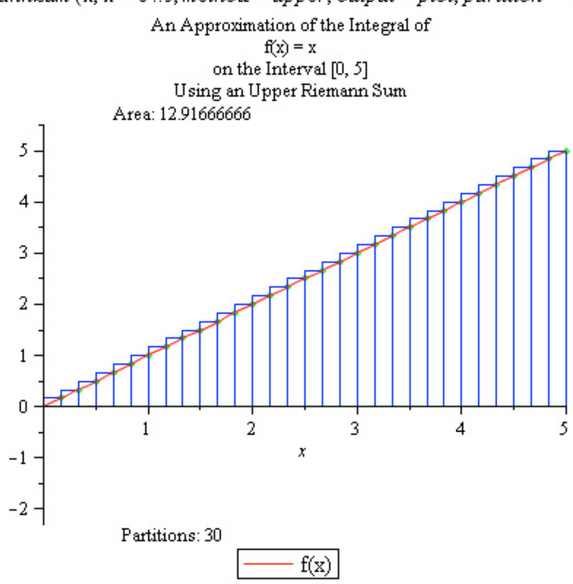


RiemannSum(*x*, x = 0 ..5, method = lower, *output* = *plot*, *partition* = 20);

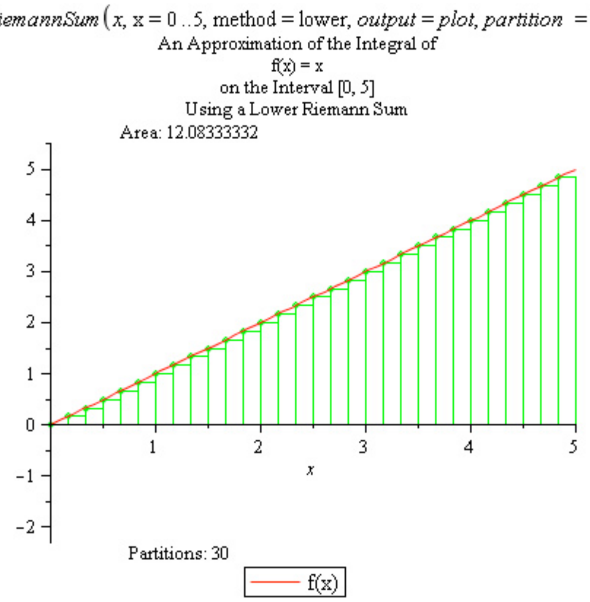


Graf pro n = 30

RiemannSum(*x*, *x* = 0 ..5, *method* = *upper*, *output* = *plot*, *partition* = 30);

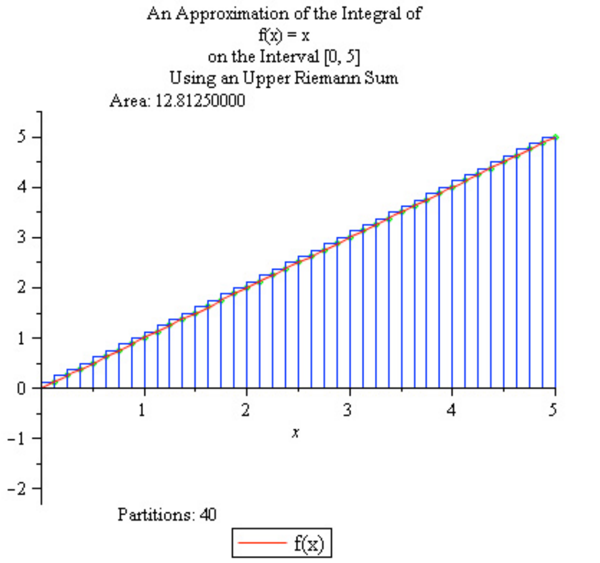


RiemannSum(*x*, x = 0 ..5, method = lower, *output* = *plot*, *partition* = 30);

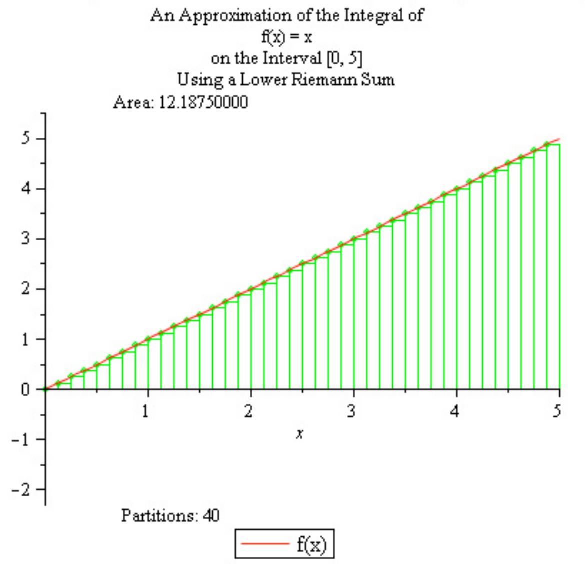


Graf pro n = 40

RiemannSum(x, x = 0 ..5, *method* = upper, *output* = *plot*, *partition* = 40);



RiemannSum(x, x = 0 ..5, *method* = lower, *output* = *plot*, *partition* = 40);



Grafy ilustruji průběh limit horních a dolních součtu, které se zvyšujícím se *n* spějí k hodnotě Riemannův integrálu pro danou funkci.